

Impact Excitation by Hot Carriers in Carbon Nanotubes

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We investigate theoretically the efficiency of intra-molecular hot carrier induced impact ionization and excitation processes in carbon nanotubes. The electron confinement and reduced screening lead to drastically enhanced excitation efficiencies over those in bulk materials. Strong excitonic coupling favors neutral excitations over ionization, while the impact mechanism populates a different set of states than that produced by photoexcitation. The excitation rate is strongly affected by optical phonon excitation and a simple scaling of the rate with the field strength and optical phonon temperature is obtained.

The excellent electrical properties and direct gap of carbon nanotubes (CNTs) offer the possibility of a unified electronic and optoelectronic technology based on this material [1]. The electroluminescence (EL) in CNTs field effect transistors has already been demonstrated, where electrons and holes were independently injected in an ambipolar device [2, 3, 4]. Recently, light emission has been observed under unipolar transport conditions in the suspended tubes [5] and from the CNT-metal contacts [4, 6]. In the suspended sample the light intensity is stronger by a factor of ~ 1000 compared to ambipolar devices. The mechanism of this enhanced EL was proposed to involve an unusually efficient intra-nanotube impact excitation process involving the hot carriers in CNTs, generated by the high local fields at the interface between the suspended and nonsuspended parts of the tube and the metal-nanotube contact. By utilizing such impact excitation process in quasi one-dimensional confined structures, high exciton densities can be achieved for probing the interactions of such boson systems and for generating ultra bright nanoscale light sources used in nanophotonics.

Here, we provide detailed theoretical study of the impact excitation in semiconducting CNTs. In low dimensional materials, the Coulomb interactions are weakly screened, and large exciton binding energies in CNTs have been predicted theoretically [7, 8, 9] and verified experimentally [10, 11, 12]. This presents an additional challenge for the computation of impact excitation rates in CNTs. We find that the impact scattering rate is very fast, up to five orders of magnitude larger than those in bulk semiconductors [13]. This can be easily rationalized since the same Coulomb interactions responsible for the large exciton binding are also responsible for the impact excitation scattering. The electron-hole correlations in the final state make the impact excitation process, on the average, twice more efficient than the impact ionization, where the created electron hole pairs are treated as independent particles. Impact excitation generates a different initial distribution of excited states than that produced by photon excitation due to the different selection rules and momentum conservation requirements.

It has recently been shown that under high bias the

energetic carriers in nanotubes excite optical and zone-boundary phonons [14, 15, 16, 17] and generate a non-equilibrium phonon distribution [18, 19], particularly when energy dissipation to the substrate is suppressed as in suspended CNTs [19]. We have calculated the effect of optical phonon excitation and found that the exciton production rate depends exponentially both on the hot phonon temperature and on the electric field, and the effect can be described by a simple scaling relation. As a result, optical phonon excitation significantly increases the EL intensity under unipolar transport conditions. The energy spectra of the excitons generated consist of several peaks corresponding to excitons with different angular momenta L : E_{11}^2 , E_{12}^3 , and E_{22}^4 , where E_{ij}^L stands for exciton with electron and hole primarily from bands i and j . The relative intensities and the overall excitation efficiencies are functions of the electric field, the optical phonon temperature, and the CNT chirality.

The impact ionization scattering rate of an electron with momentum and energy $(k_1, \varepsilon_{k_1}^c)$ scattered to state $(k_1 - q, \varepsilon_{k_1-q}^c)$ plus an e-h pair with a hole $(-k_2, -\varepsilon_{k_2}^v)$ and an electron $(k_2 + q, \varepsilon_{k_2+q}^c)$ is given by [20]:

$$W_{k_1, k_1-q} = \frac{2\pi}{\hbar} \sum_{k_2} |M_\nu|^2 \delta(\varepsilon_{k_1}^c - \varepsilon_{k_2}^v - \varepsilon_{k_1-q}^c - \varepsilon_{k_2+q}^c) |M_\nu|^2 = |M_\nu^d - M_\nu^e|^2 + |M_\nu^d|^2 + |M_\nu^e|^2 \quad (1)$$

where $\nu = k_1 k_2 q$, $M_\nu^d = J(k_1 k_2, k_2 + q k_1 - q)$ and $M_\nu^e = J(k_1 k_2, k_1 - q k_2 + q)$ are the direct and the exchange integrals:

$$J(k_1 k_2, k_1 - q k_2 + q) = \int d\vec{r}_1 d\vec{r}_2 \Psi_{ck_1}^*(\vec{r}_1) \Psi_{vk_2}^*(\vec{r}_2) \Psi_{ck_1-q}(\vec{r}_1) \Psi_{ck_2+q}(\vec{r}_2) V(\vec{r}_1, \vec{r}_2) \quad (2)$$

$V(\vec{r}_1, \vec{r}_2)$ is the Coulomb potential screened by the dielectric environment of the nanotube [9] and $\Psi_c(\vec{r})$ and $\Psi_v^*(\vec{r})$ are the electron and the hole wavefunctions, given by a π -orbital tight-binding model with hopping matrix element $t = 3.0$ eV. In Eq. (1) we neglect e-h pair occupation in the initial state and, hence, processes where electron can absorb an e-h pair.

To include the effect of e-h correlation in the final state we follow [21] and show that the impact excita-

tion scattering rates due to the production of singlet (S) and triplet (T) excitons are given by:

$$\begin{aligned}
 W_{k_1 k_1 - q}^S &= \frac{\pi}{\hbar} \sum_{\mu} \left| \tilde{M}_{k_1 q \mu}^S \right|^2 \delta(\varepsilon_{k_1}^c - \varepsilon_{k_1 - q}^c - E_{q\mu}^S) \\
 W_{k_1 k_1 - q}^T &= \frac{\pi}{\hbar} \sum_{\mu} \left| \tilde{M}_{k_1 q \mu}^T \right|^2 \delta(\varepsilon_{k_1}^c - \varepsilon_{k_1 - q}^c - E_{q\mu}^T) \\
 \left| \tilde{M}_{k_1 q \mu}^S \right|^2 &= \left| \tilde{M}_{k_1 q \mu}^d - \tilde{M}_{k_1 q \mu}^e \right|^2 + \left| \tilde{M}_{k_1 q \mu}^d \right|^2 \\
 \left| \tilde{M}_{k_1 q \mu}^T \right|^2 &= \left| \tilde{M}_{k_1 q \mu}^d - \tilde{M}_{k_1 q \mu}^e \right|^2 + \left| \tilde{M}_{k_1 q \mu}^d \right|^2 + 2 \left| \tilde{M}_{k_1 q \mu}^e \right|^2 \\
 \tilde{M}_{k_1 q \mu}^d &= \sum_{k_2} A_{k_2 q}^{\mu} M_{k_1 k_2 q}^d \\
 \tilde{M}_{k_1 q \mu}^e &= \sum_{k_2} A_{k_2 q}^{\mu} M_{k_1 k_2 q}^e
 \end{aligned} \quad (3)$$

where $E_{q\mu}^S$ and $E_{q\mu}^T$ are the singlet and triplet exciton transition energies [22], A_{kq}^{μ} is the solution of the Bethe-Salpeter equation [8, 9] for the exciton two-particle wavefunction:

$$\Phi_q^{\mu}(\vec{r}_1, \vec{r}_2) = \sum_k A_{kq}^{\mu} \Psi_{ck+q}(\vec{r}_1) \Psi_{vk}^*(\vec{r}_2). \quad (4)$$

We model the screening of the Coulomb interaction by the dielectric constant of the medium embedding the nanotube [9], both for the Bethe-Salpeter equation kernel and the impact ionization Coulomb potential Eq. (2). The CNT diameter dependence of exciton binding energies obtained by two-photon fluorescence excitation spectroscopy [23] agree very well with our model calculations [9] by choosing $\epsilon = 3.3$, a value which we use in the rest of the paper.

The impact ionization and impact excitation scattering rates for a tube of diameter $d = 2$ nm are shown in Fig. 1a and 1b respectively. This is a typical diameter of tubes used in opto-electronic applications and grown by chemical vapor deposition. The impact excitation rates in CNTs are roughly twice as large as the impact ionization rates due to the e-h confinement in the final state. To understand the structure of the impact ionization scattering rates in Fig. 1a, we need to consider the implications of energy and momentum conservation. The momentum has two components: a discrete angular momentum, which labels the bands, and a linear momentum along the tube axis. The CNT bands are non-parabolic and an angular momentum conservation law plays a crucial role in determining the threshold energy of the impact excitation [24]. As seen from Fig. 1c, angular momentum conservation allows electrons in the third, e_3 , and the fourth, e_4 , bands to undergo impact ionization scattering by creating electron hole pairs in the first and the second bands respectively: $e_3^4 = e_2^2 + e_1^1 + h_1^{-1}$ and $e_4^5 = e_1^1 + e_2^2 + h_2^2$, where the upper subscript L_i is the angular momentum in units of $2/3d$. In addition, angular momentum determines the band edge energy Δ_i . In the large d limit, $\Delta_i = \hbar v_F L_i$,

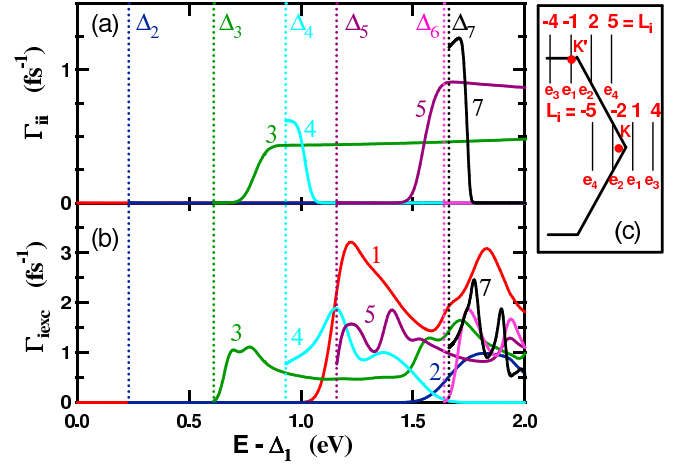


FIG. 1: (a) Impact ionization rates and (b) impact excitation rates for a (25,0) nanotube ($d = 2.0$ nm) as a function of energy (measured from the bottom of the first conduction band Δ_1), for the first seven conduction bands shown in red, blue, green, cyan, magenta, light red, and black in increasing order of the band index. The vertical dashed lines correspond to the bottoms of the conduction bands Δ_i ($i = 2, 3, \dots, 7$) relative to Δ_1 . (c) The vertical lines are allowed k-points of zig-zag tube with $mod(n, 3) = 1$ for the first four doubly degenerate bands $e_i^{L_i}$. The angular momenta L_i , in units of $2/3d$, measures a minimum distance from the K (K') point of graphene to the 1D lines of allowed k-points of a nanotube.

where v_F is the Fermi velocity of graphene. Therefore, in this limit, the energy and momentum conservations are simultaneously satisfied for the electrons at the bottom of the third and the fourth bands. However, for finite diameter tubes: $\Delta_3 < \Delta_2 + 2\Delta_1$ in $mod(n - m, 3) = 1$ chirality and $\Delta_4 < \Delta_1 + 2\Delta_2$ in $mod(n - m, 3) = -1$ chirality. This pushes the lowest impact ionization threshold energy E_{th3} of a (25, 0) CNT in the third band at energies higher than its bottom Δ_3 , i. e. $E_{th3} > \Delta_3$, and a power law dependence is obtained $\Gamma_{ii3} \propto (E - E_{th3})^\alpha$ with $\alpha \approx 2$. Whereas at the bottom of the fourth band Δ_4 , energy and momentum conservation laws are simultaneously satisfied such that the threshold energy E_{th4} coincides with the bottom of the band Δ_4 , i. e. $E_{th4} = \Delta_4$. The energy dependence of the scattering rate at the onset in the fourth band is similar to that of a step function $\Gamma_{ii4} \propto \Theta(E - E_{th4})$. In a (26, 0) tube the impact ionization threshold in the third band coincides with Δ_3 , i. e. $E_{th3} = \Delta_3$, and it is higher than Δ_4 in the fourth band, i. e. $E_{th4} > \Delta_4$. For impact ionization to take place in the first (second) band via the decay channel $e_1^1 = e_1^1 + e_1^1 + h_1^{-1}$ ($e_2^2 = e_2^2 + e_1^1 + h_1^{-1}$), the threshold energy must be at least $E_{th1} \geq 3\Delta_1$ ($E_{th2} \geq \Delta_2 + 2\Delta_1$) and momentum conservation along the 1D wavevector in non-parabolic bands brings the threshold for the first (second) bands to energies beyond the scale of Fig. 1a [24].

In impact excitation, the exciton state formed has an

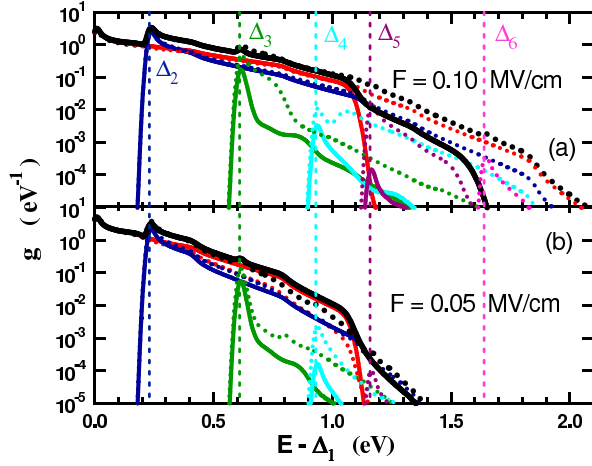


FIG. 2: Electron distribution function in a (25,0) nanotube at applied electric field of (a) $F = 0.1$ MV/cm and (b) $F = 0.05$ MV/cm as a function of energy in the first six conduction bands shown in red, blue, green, cyan, magenta, and light red, respectively. Black curves are the total distributions. In (a) solid curves use impact excitation and dotted curves use impact ionization scattering rates with phonons at $T = 300$ K. In (b) we use impact excitation scattering, solid curves use phonon scattering at $T = 300$ K and dotted curves use acoustic phonon scattering at $T_{ac} = 300$ K and optical phonon scattering at $T_{op} = 1500$ K.

effective mass enhanced by 30% over the free particle value ($m_e + m_h$), due to the Coulomb interaction [25]. (We find that the magnitude of the mass enhancement depends on the strength of Coulomb interaction as $1/\epsilon$, independent of tube chirality.) This mass enhancement lowers the threshold energy. Indeed, in the limit of the infinite effective mass, the exciton dispersion is flat, and therefore, energy and angular momentum conservations alone would determine the impact excitation threshold. Thus, the impact excitation thresholds of the first two conduction bands (Fig. 1b) are significantly reduced from the impact ionization values [24]. At the onset of impact excitation, the third and the fourth band electrons can decay as $e_3^4 = e_2^2 + E_{11}^2$ or $e_3^4 = e_1^1 + E_{12}^3$ and $e_4^5 = e_2^2 + E_{12}^3$ or $e_4^5 = e_1^1 + E_{22}^4$. The relative efficiencies of the two decay channels depend on tube chirality and electron energy.

In electro-optical applications, we need to know the effect of impact excitation scattering on the excited state production rate. This depends on the hot carrier distribution function, which we calculate by solving the steady-state multi-band Boltzmann equation in the presence of an electric field, electron-phonon scattering modeled as in [17], and either impact ionization Eq. (1), or impact excitation scattering Eq. (3). The results are shown in Fig. 2. At energies above the optical phonon energy of about 180 meV, the carrier distribution is determined by the field F and the optical phonon mean free path λ_{op} . The probability that an electron is accelerated to an energy $E = eF\lambda$ over a length λ without being scattered by

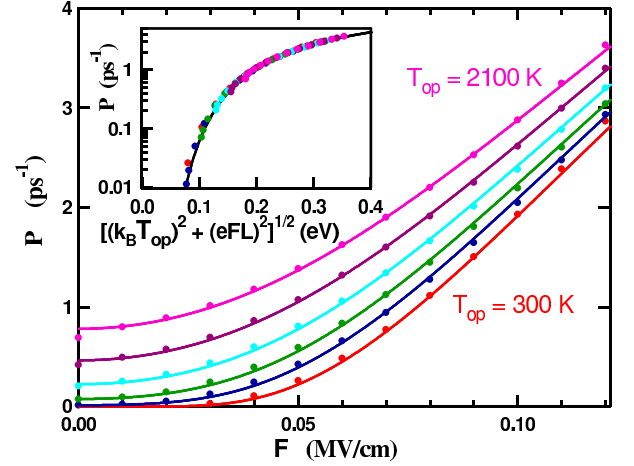


FIG. 3: Exciton production rate P per unit carrier in a (25,0) tube as function of the electric field for acoustic phonon scattering at $T_{ac} = 300$ K and different optical phonon temperatures (from bottom to top) $T_{op} = 300$ K (red), 900 K (blue), 1200 K (green), 1500 K (cyan), 1800 K (magenta), 2100 K (light red) along with the best fit to Eq. (5) for $\lambda_{op} = 25$ nm, $E_t = 0.57$ eV and $P_0 = 18$ ps $^{-1}$. The inset shows scaling of all the calculations with the effective temperature in accord with Eq. (5).

an optical phonon is $\exp(-\lambda/\lambda_{op})$. Therefore, the carrier distribution g , for energies above the optical phonon and below the impact excitation (ionization) threshold, is expected to follow $g(E) \propto \exp(-E/eF\lambda_{op})$, as seen in Fig. 2. Above the impact excitation (ionization) threshold, the scattering is an order of magnitude larger and so is the slope of $\ln(g(E))$ versus E , as in Fig. 2. The higher threshold for impact ionization results in a “hotter” carrier distribution than in the case of impact excitation (Fig. 2a).

The non-equilibrium phonon effect on the carrier distribution is shown in Fig. 2b. Although at optical phonon temperature $T_{op} = 1500$ K the phonon scattering rate is a factor 1.7 larger than at room temperature, the slope of $\ln(g(E))$ versus E , for energies below the impact excitation threshold, is similar to the $T_{op} = 300$ K case. On the other hand, the hot carrier tail above the impact excitation threshold significantly increases with T_{op} , which in turn increases the exciton production rate.

The exciton production rate itself is given by the product of the carrier distribution, as in Fig. 2, and the impact excitation scattering rate, as in Fig. 1b, and it is shown in Fig. 3. We find that the rate P can be well fitted by the following equation:

$$P = P_0 \exp \left(- \frac{E_t}{\sqrt{(k_B T_{op})^2 + (eF\lambda_{op})^2}} \right), \quad (5)$$

where E_t is the impact excitation threshold energy and P_0 is a constant. The form of Eq. (5) resembles the Boltzmann distribution with an effective temperature in the

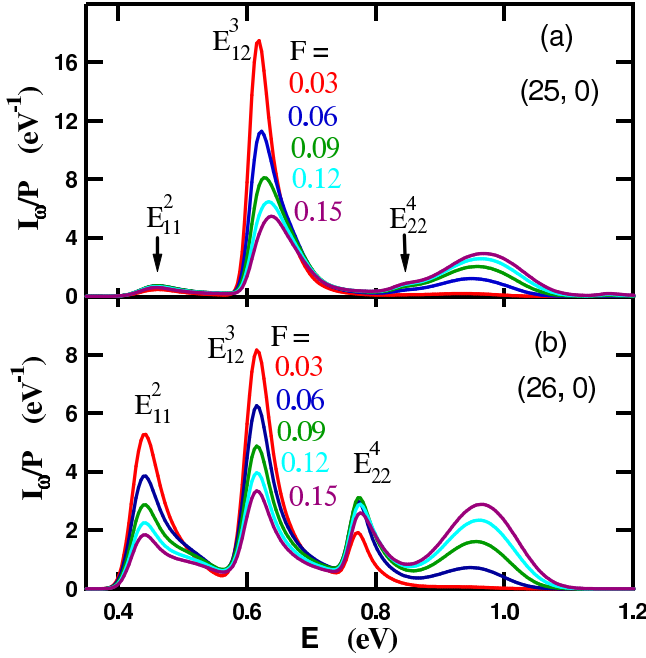


FIG. 4: Energy distribution I_ω of excitons produced in the impact excitation process in (a) a (25,0) tube and (b) a (26,0) tube with phonons at $T = 300$ K and different applied electric fields F in MV/cm: 0.03 - red, 0.06 - blue, 0.09 - green, 0.12 - cyan, 0.15 - magenta, and normalized to the total production rate $P = \int I_\omega d\omega$.

presence of the field. The inset of Fig. 3 shows an excellent scaling of the exciton production rate with the effective temperature. The diameter dependence of the fit parameters in Eq. (5) for $1 \text{ nm} < d < 2 \text{ nm}$ is very simple: $E_{th} \approx 1.22 \text{ (eV nm)}/d$, $\lambda_{op} \approx 14 \times d$, while $P_0 \approx 18 \text{ ps}^{-1}$ is nearly diameter independent.

Excitons in nanotubes can be created either by optical pumping as in photoluminescence experiments, or by electrical excitation as in electroluminescence. In the photoluminescence case, the energy of the created excitons follows the photon energy of the excitation light source, and the angular momentum distribution is determined by the dipole selection rules. Whereas in the impact excitation process, the energy distribution of the generated excitons depends on the applied bias and it is not subject to the same selection rules. In Fig. 4 we plot the distribution function of the initially created excitons at different fields. The distribution has several distinct peaks, which we identify by analyzing the contributions from excitons with different angular momenta. At the onset of impact excitation, the third and the fourth band electrons generate the low energy excitons with finite angular momenta: E_{11}^2 , E_{12}^3 , E_{22}^4 , whereas at higher bias electrons in the first band produce unbound excitons of zero angular momentum, which contribute significantly to a broad continuum at energy of about 0.9 eV in Fig. 4.

We conclude that CNTs and other 1D systems are

characterized by unusually high impact excitation rates and can be used as efficient exciton sources to be employed in fundamental studies of interacting boson systems at high densities and in opto-electronic device applications.

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